TRINITY COLLEGE

# Mathematics Methods Units 3,4
# Test 6 2019

Calculator Assumed
**Sample Proportions & Confidence Intervals**

**STUDENT'S NAME** MARKING KEY

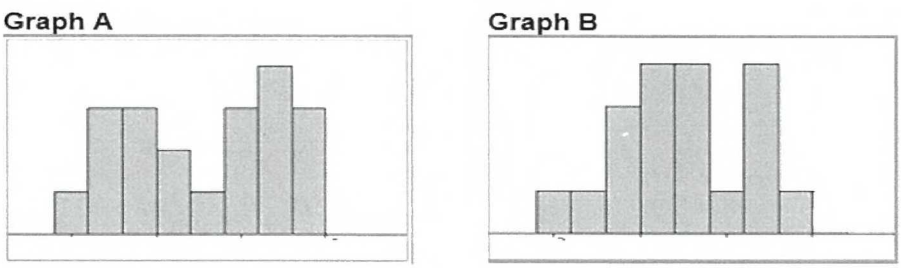
**DATE:** Thursday 5th September **TIME:** 50 minutes **MARKS:** 43

**INSTRUCTIONS:**
Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

1. (2 marks)

A random sample is simulated from a uniform distribution. The following two graphs represent possible sampling distributions with one representing a sample of size 20 and the other a sample size of 100.




Which graph would best represent the sample of 100? Give a reason for your answer.

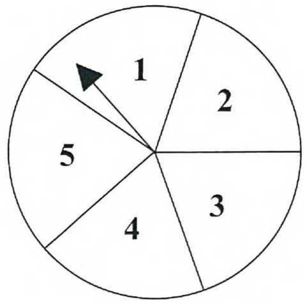
GRAPH A, As it is a uniform distribution the larger sample should lead to a more evenly spread distribution.

or

GRAPH B, as we are taking samples the sample proportion $\hat{p}$ will tend toward a normal dist. as $n$ increases.

2. (6 marks)

The spinner shown has each region equally likely to occur.




Suppose we wish to investigate the likelihood of achieving an even number on a spin. The spinner is spun 120 times resulting in 53 even numbers.

(a) State the value of $\hat{p}$ for this sample. [1]

$$\hat{p} = \frac{53}{120}$$

$$= 0.44167$$

(b) Calculate the value of $\dfrac{\hat{p}-p}{\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}}$ using your value of $\hat{p}$ from (a). [3]

$p = \frac{2}{5}$

$$\frac{\frac{53}{120} - \frac{2}{5}}{\sqrt{\dfrac{\frac{53}{120}\left(1-\frac{53}{120}\right)}{120}}}$$

$$= 0.9191$$

(c) Sample proportions for multiple samples of 120 spins are recorded. The statistic in (b) is then calculated for each, describe the distribution of these values. [2]

Normal $\mu = 0$, $\sigma = 1$

or

Standard Normal Distribution

or

$X \sim N(0,1)$

3. (7 marks)

To estimate the proportion of Western Australian high school students that play the piano, a sample of 200 students is taken.

Of the 200 students in the sample, it was found that 178 students played the piano.

(a) Use this sample to estimate the:

(i) true proportion of Western Australian high school students that play the piano, [1]

$$\hat{p} = \frac{178}{200} = 0.89$$

(ii) standard deviation of sample proportions of Western Australian high school students that play the piano. [2]

$$\sigma = \sqrt{\frac{0.89(1-0.89)}{200}}$$

$$= 0.02212$$

(b) If this sample was taken from a high school that specialises in music, discuss with reasons the accuracy of the estimate of the true proportion of Western Australian high school students that play the piano. [2]

Inaccurate/biased sample

We would expect that the true proportion would be lower.

The true proportion of Western Australian high school students that play the piano is 0.29.

(c) If the sample was taken from an ordinary high school, suggest two possible random sampling techniques which could have been used. [2]

- Use a random number generator to select specific students.
- Randomly select names from a hat.

4. (8 marks)

In a randomly selected sample of 50 residents from a city, 16 were born overseas.

(a) Use this sample to determine $\hat{p}$, the sample proportion of overseas born residents in this city. [1]

$$\frac{16}{50} = 0.32$$

(b) Use this sample to provide a 90% confidence interval for $p$. [3]

$Z = 1.645$

$$E = 1.645 \times \sqrt{\frac{0.32(1-0.32)}{50}}$$

$$= 0.1085$$

$$90\% \text{ C.I} = 0.2115 \leq p \leq 0.4285$$

(c) In a second sample of 80 residents, 32 were overseas-born. Use your answer in (b) to determine with reasons if this sample statistically has a higher proportion of overseas born residents. [2]

$$\frac{32}{80} = 0.4$$

This is within the 90% C.I in (b) ∴ we cannot say it has a statistically higher proportion.

(d) Using the sample proportion of the survey from part (a), determine a smallest sample size that will give at most a 10% margin of error with a confidence of 90%. [2]

$$0.1 = 1.645\sqrt{\frac{0.32(1-0.32)}{n}}$$

$$n = 58.87$$

$$\sim 59$$

Sample size required is 59 residents.

5. (7 marks)

In a sample of $n$ houses, 27 were found to have smoke detectors. Using this sample, a $c$ % confidence interval for the true proportion of houses with smoke detectors was $0.26 \leq p \leq 0.46$ .

(a) Calculate the value of $n$ . [2]

$$p = \frac{0.26 + 0.46}{2}$$

$$p = 0.36$$

$$\frac{27}{n} = 0.36$$

$$n = 75$$

(b) Calculate the value of $c$ . [5]

"sample n for CI"

error is 0.1

$n = 75$

$z = ?$

$p = 0.36$

$e = 0.1$

$z = 1.8042196$

"z for CI"

$z = 1.8042196$

$c = 0.9288$

$\therefore$ C.I = 93 %

or

$$0.46 = 0.36 + z\sqrt{\frac{0.36(1-0.36)}{75}}$$

$$z = 1.8042$$

$$P(-1.8042 < z < 1.8042) = 93\%$$

6. (4 marks)

It is known that $p$ % of students in a certain state of Australia are international students. 50 samples of 100 students are taken and the proportions of international students were calculated. The sampling distribution of the sample proportions has a standard deviation of 0.035. Determine with reasons, a reasonable value of $p$.

$$0.035 = \sqrt{\frac{p(1-p)}{100}}$$

$$p = 0.1429 \text{ or } 0.8571$$

$p = 0.1429$ is the reasonable value for $p$ as we are referring to international students.

7. (5 marks)

A recent survey indicated that 56% of Australians are in favour of Australia becoming a Republic.

(a) Assuming that this proportion represents the population proportion, how many people should be surveyed to ensure that the 95% confidence interval for the survey has a width of less than 2%? [3]

error = 0.01

$$0.01 = 1.96\sqrt{\frac{0.56(1-0.56)}{n}}$$

$n = 9465.35$

$n = 9466$

(b) Assuming that the margin of error remains the same, what would be the effect on the size of the sample if:

(i) the population proportion of those in favour was much higher? [1]

As $p(1-p)$ will be smaller the number will decrease.

(ii) the survey required a 99% level of confidence? [1]

As the confidence level increases $n$ will also increase.

8. (4 marks)

When taking samples of size 400 from a population, it was found that 6% of samples had a proportion that was more than 0.03 above the population proportion. Determine the population proportion.

z score for $P(z > k) = 0.06$

$k = 1.5548$

Solve $1.5548 = \dfrac{\hat{p} - p}{\sqrt{\dfrac{p(1-p)}{400}}}$

$1.5548 = \dfrac{0.03}{\sqrt{\dfrac{p(1-p)}{400}}}$

$p = 0.182$ or $p = 0.818$

or

6% 0.03 above $\hat{p}$, 6% 0.03 below $\rightarrow$ 88% C.I

Error $= 0.03$

Solve $0.03 = 1.5548\sqrt{\dfrac{p(1-p)}{400}}$

$p = 0.182$ or $p = 0.818$